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Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, June 2016
(2008 Scheme)
08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time : 3 Hours Max. Marks : 100

***Instructions** : Answer **all** questions from Part – **A** and **one** full question from **each** Module of Part – **B**.*

PART – A

1. Prove that an analytic function with constant modulus is a constant.

2. Find the analytic function $f(z) = u(x, y) + iv(x, y)$ whose real part is $y + e^x \cos y$.

3. Find the constant 'a' so that the function e^{ax} cosy is harmonic.

4. Find the fixed points of $w = \frac{5z+4}{z+5}$.

5. Show that if f(z) is analytic in the region bounded by two simple closed curves C_1 and C_2 then $\oint_{c_1} f(z)\,dz = \oint_{c_2} f(z)\,dz$.

6. Define (i) Singular point (ii) Pole (iii) Removable singularity (iv) Essential singularity.

7. Find the residue at $z = 0$ of $\frac{1+e^z}{z \cos z + \sin z}$.

8. Write a note on errors in numerical computations.

P.T.O.

9. Find a positive root of $x^4 - x = 10$ correct to 3 decimal places using Newton Raphson Method.

10. Using Newton's backward interpolation formula, find θ at $x = 84$ from the following table

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

(10×4=40 Marks)

PART – B

Module – 1

11. a) Show that the function f(z) is discontinuous at the origin, z = 0, given that

$$f(z) = \begin{cases} \dfrac{xy(x-2y)}{x^3+y^3}, & \text{when } z \neq 0 \\ 0, & \text{when } z = 0 \end{cases}$$

b) If $f(z) = u + iv$ is an analytic function of z; then show that $\nabla^2 u^p = p(p-1)\,u^{p-2}\,|f'(z)|^2$.

c) Find the image of $|z - 2i| = 2$ under the transformation $w = 1/z$.

12. a) If $w = u + iv$ is an analytic function and $v = x^2 - y^2 + \dfrac{x}{x^2+y^2}$, find u.

b) Find the analytic function $f(z) = u + iv$, given that $u - 2v = e^x\,[\cos y - \sin y]$.

c) Define cross ratio and prove that the cross ratio remains invariant under Bilinear Transformation.

Module – 2

13. a) Evaluate $\int_0^{1+i} (x - y + ix)^2\,dz$ along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1 + i.

b) Obtain Taylor's series to represent the function $\dfrac{z^2-1}{(z+2)(z+3)}$ in the region $|z| < 2$.

c) Evaluate $\int_C \dfrac{3z^2+z}{z^2-1}\,dz$ where C is the circle $|z - 1| = 1$.

14. a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 - 4\cos\theta} d\theta$.

b) Evaluate $\int_0^{\infty} \frac{dx}{1+x^2}$.

Module – 3

15. a) Find a real root of $f(x) = x^3 - x - 1 = 0$ using Bisection method.

b) Find the polynomial f(x) by using Lagrange's formula and hence find f(3) for

x	0	1	2	5
f(x)	2	3	12	147

c) Solve the equations $4x + 2y + z = 14$, $x + 5y - z = 10$, $x + y + 8z = 20$ by Gauss-Siedel Method.

16. a) Using Newton's forward interpolation formula, find the polynomial f(x) satisfying the following data. Hence evaluate y at x = 5.

x	4	6	8	10
y	1	3	8	10

b) Using Simpson's one-third rule evaluate $\int_0^1 x e^x \, dx$ taking 4 intervals. Compare your result with actual value

c) Solve $\frac{dy}{dx} = x + y$; $y(0) = 1$ by Taylor series method. Find the values of y at x= 0.1 and x = 0.2. **(3×20=60 Marks)**

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